IOPscience iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

Periodic ferrite–semiconductor layered composite with negative index of refraction

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

2007 J. Phys.: Condens. Matter 19 026211

(http://iopscience.iop.org/0953-8984/19/2/026211)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 129.252.86.83
The article was downloaded on 28/05/2010 at 15:20

Please note that terms and conditions apply.

Institute of Physics Publishing — Journal of Physics: Condensed Matter

J. Phys.: Condens. Matter **19** (2007) 026211 (8pp) doi:10.1088/0953-8984/19/2/026211

Periodic ferrite–semiconductor layered composite with negative index of refraction

Rui-Xin Wu[1], **Tianen Zhao**[1] **and John Q Xiao**[2]

[1] Department of Electronic Sciences and Engineering, Nanjing University, Nanjing 210093, People's Republic of China

[2] Department of Physics and Astronomy, University of Delaware, Newark, DE 19716, USA

E-mail: rxwu@nju.edu.cn

Received 25 July 2006, in final form 12 November 2006
Published 15 December 2006
Online at stacks.iop.org/JPhysCM/19/026211

Abstract

We have studied the index of refraction of periodic layered semiconductor–ferrite composite. Both the transmission matrix analysis and full-wave simulations confirm the existence of a negative index of refraction in the composite. We find that the magnitude of the negative index of refraction and its frequency range depend on the ratio of the semiconductor and ferrite layer thicknesses and the doping content of the semiconductor. At the long-wave approximation, we have obtained the explicit expression of the effective index of refraction, effective permittivity and effective permeability. The result shows that it is possible to fabricate a uniform negative index material with the layered composite.

Negative index materials (NIMs) have attracted much attention since the observation of negative refraction in metamaterials consisting of periodic arrays of metallic rings and rods [1]. NIMs have several extraordinary properties [2] and many potential applications [3, 4]. For example, Pendry [4] predicted that a lossless negative index slab can act as a perfect lens, which overcomes the resolution limitation in conventional imaging systems. In addition to metamaterials, negative refraction has also been found in the other materials. Grbic and Eleftheriades [5] have demonstrated that a specially designed planar transmission line has negative refraction, and can be used as a left-handed lens. Parimi *et al* [6] observed negative refraction in microwave photonic crystals. Chen *et al* [7] found that negative refraction also exists in randomized patterns. All NIMs mentioned above are composite structures, and they rigorously cannot be considered as uniform materials, since the size of structure elements are usually greater than 1/10 of the wavelength. For example, the size of the unit cell of the metamaterial is about 5 mm in the X band [8], which is over 1/6 of the wavelength. In addition, certain field polarization is required in order to exhibit the negative refraction. These requirements encumber their practical applications. Consider that the rings and rods produce negative permittivity ε and negative permeability μ, respectively [9]; the metamaterials are similar to multilayers with alternating negative ε and negative μ layers, which could be realized with natural materials. It is well known that some natural materials have inherent negative ε or negative μ. For example, certain ferrites have negative permeability in microwave frequencies, and conducting materials have negative permittivity below the plasma frequency. Recently, studies show the possibility of making NIM composites with natural materials. Ueda and Tsutsumi [10] have studied the stacked ferrite–dielectric system in a rectangular waveguide and experimentally demonstrated the existence of a pass band in the cut-off frequency region. Wu [11] has demonstrated the existence of a negative index in a periodic layered ferrite–metal film system. Pimenov *et al* found negative refraction in ferromagnet–superconductor superlattices [12]. However, the use of metallic or superconducting materials may excite surface waves at the interface, which in turn change the electromagnetic characters of the composite. Meanwhile, the thickness of metallic layers should be very small (about several nanometres), which has caused some difficulties in the deposition of ferrite–metal multilayers. To overcome these problems, in this paper we propose a new configuration—the layered ferrite–semiconductor composite. We will show that the composite has a negative refraction index, but only with negative permeability, and the magnitude of the negative refraction index and its frequency bandwidth can be tuned by the thickness ratio of the semiconductor and the ferrite and the doping concentration of the semiconductor.

The proposed composite consists of alternating layered ferrite and semiconductor, as shown in figure 1(a). The composite is periodic along the z-axis. In order to obtain the effective index of refraction, the layered composite should satisfy certain conditions. It has been demonstrated that the composite has an equivalent single layer if the thickness of each layer is much smaller than the wavelength, or the multilayers have symmetrical configurations [13, 14]. Without losing the generality, here we choose the unit cell with a symmetrical configuration, i.e. semiconductor–ferrite–semiconductor or vice versa. Since each layer is a uniform medium, the composite can be simulated by a series of transmission lines, as shown in figure 1(b), where the voltage V and current I represent field E and H, respectively [15]. According to the theory of transmission lines, the input–output of the voltage and current for the unit cell can be expressed as a product of a transmission matrix of each layer, that is

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \overline{\overline{T}}_1 \overline{\overline{T}}_2 \overline{\overline{T}}_1 \begin{pmatrix} V_2 \\ I_2 \end{pmatrix} = \overline{\overline{T}} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix} \tag{1}$$

and

$$\overline{\overline{T}}_m = \begin{pmatrix} \cos(k_m d_m) & \mathrm{j} Z_m \sin(k_m d_m) \\ \mathrm{j} Z_m^{-1} \sin(k_m d_m) & \cos(k_m d_m) \end{pmatrix} \tag{2}$$

where $m = 1, 2$, k_m, Z_m, and d_m are the propagation constant, characteristic impedance and thickness of the layers, respectively. Considering that the composite is periodic, the V and I should satisfy the Floquet theorem [15], which means that the difference of V and I between port 1 and port 2 in the transmission line should be a constant $\mathrm{e}^{\pm \mathrm{j}\beta d}$, where d is the thickness of the unit cell and β is the complex propagation constant of the composite. Then, equation (1) becomes

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \overline{\overline{T}} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix} = \mathrm{e}^{\pm \mathrm{j}\beta d} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix}. \tag{3}$$

Equation (3) shows that $\mathrm{e}^{\pm \mathrm{j}\beta d}$ is the eigenvalue of the transmission matrix $\overline{\overline{T}}$. It is easy to prove that the characteristic equation of $\overline{\overline{T}}$ satisfies

$$\lambda^2 - (T_{11} + T_{22})\lambda + 1 = 0 \tag{4}$$

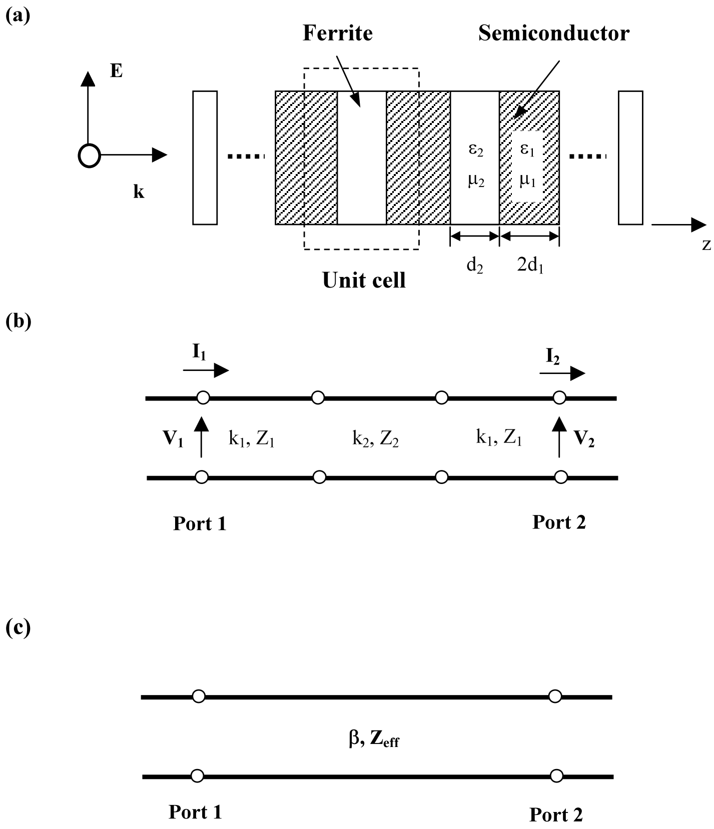

Figure 1. (a) Schematic diagrams of a periodic layered semiconductor–ferrite composite with a plane wave normally incident. (b) Each layer is equivalent to a transmission line, and the unit cell is thus a series of transmission lines. (c) For a symmetrical configuration of the unit cell, the series transmission lines are represented by a single transmission line.

where T_{11} and T_{22} are the matrix elements of $\overline{\overline{T}}$. From equation (4), we have dispersion equation $\cos(\beta d) = (T_{11} + T_{22})/2$. With the expressions of T_{11} and T_{22} derived from equations (1) and (2), we obtain the explicit expression for the dispersion equation,

$$\cos(\beta d) = \cos(2k_1 d_1)\cos(k_2 d_2) - \frac{1}{2}\left(\frac{Z_1}{Z_2} + \frac{Z_2}{Z_1}\right)\sin(2k_1 d_1)\sin(k_2 d_2), \quad (5)$$

which is similar to the dispersion equation for photonic bandgap materials [16]. Since the configuration is symmetrical, the unit cell can be represented by an equivalent single layer which is described by a single transmission line, as shown in figure 1(c). The effective refraction index is thus defined as

$$n_{\text{eff}} \equiv c/v_\varphi = \beta c/\omega, \quad (6)$$

where c is the speed of light, and v_φ is the wave velocity. Further, if the length of the unit cell is much smaller than the wavelength, the composite can be considered as a uniform medium. Hence, we have the effective characteristic impedance of the unit cell:

$$Z_{\text{eff}}^2 = Z_1^2 \frac{Z_1 Z_2 \sin(2k_1 d_1)\cos(k_2 d_2) + (Z_2^2\cos^2(k_1 d_1) - Z_1^2\sin^2(k_1 d_1))\sin(k_2 d_2)}{Z_1 Z_2 \sin(2k_1 d_1)\cos(k_2 d_2) + (Z_1^2\cos^2(k_1 d_1) - Z_2^2\sin^2(k_1 d_1))\sin(k_2 d_2)}. \quad (7)$$

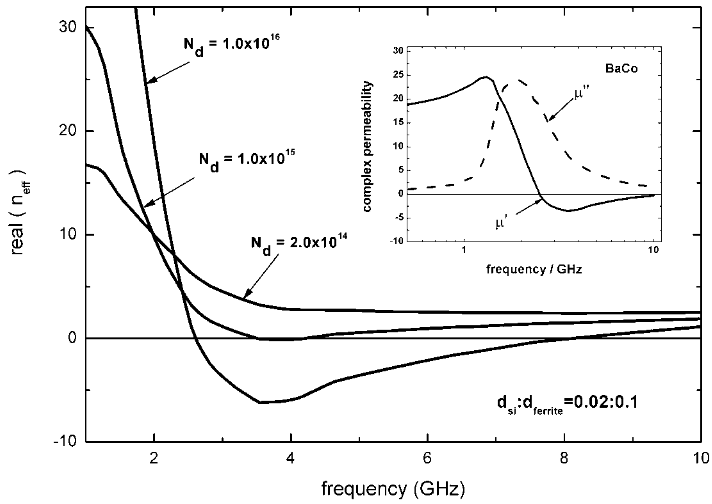

Figure 2. The index of refraction of the composite with different doping content as a function of frequency computed with equations (5) and (6). The thicknesses d_1 and d_2 are 0.02 and 0.1 mm, respectively. The inset is the frequency dependence of complex permeability of ferrite $Ba_3Co_2Fe_{24}O_{41}$.

With equations (6) and (7), we obtain the effective permittivity and effective permeability:

$$\varepsilon_{\mathrm{eff}} = \frac{n_{\mathrm{eff}}}{Z_{\mathrm{eff}}} \qquad \mu_{\mathrm{eff}} = n_{\mathrm{eff}} Z_{\mathrm{eff}}. \tag{8}$$

For the ferrite–semiconductor composite, a negative index needs a ferrite with negative permeability, since the permittivity of the semiconductor and ferrite are always positive. It is well known that some ferrites have a negative μ_r in the vicinity of the natural ferromagnetic resonant frequency. For example, a negative μ_r is found in the barium-cobalt ($Ba_3Co_2Fe_{24}O_{41}$) ferrites [11]. To demonstrate that the proposed composite has a negative effective refraction index, we use BaCo as magnetic layers. Its complex permittivity is about $12(1–10^{-3}\mathrm{j})$, and the complex permeability is frequency dependent, which is shown in the inset of figure 2. Doped silicon is selected as the dielectric layers. The permittivity of the doped silicon is a complex number. The real part of the complex permittivity ε_{r1} is about 11.9, the permittivity of intrinsic Si. The imaginary part ε_{r2} is mainly dependent on the conductivity σ with the form $\varepsilon_{r2} = \sigma/\omega\varepsilon_0$ [17]. The conductivity varies with doping content N_D. Reference [18] gives the variation of conductivity with the doping content of silicon and other commonly used semiconductors. With the unit cell configuration shown in figure 1(a) and taking the thickness of the BaCo and doped silicon to be 0.1 and 0.02 mm, respectively, we first calculated the effective refraction index, n_{eff}, using equations (5) and (6). The results are shown in figure 2. We can find that a negative effective index of refraction appears at certain frequencies when the doping content is over 10^{15} cm^{-3}. Comparing the curves of n_{eff} and the permeability spectrum of BaCo in figure 2, the frequencies with negative n_{eff} are within the range where BaCo has negative permeability. The frequency range for negative n_{eff} is closely related to the doping content. When the doping content is increased, the bandwidth of negative n_{eff} becomes wider. Doping content greatly affects the existence and the frequency bandwidth of negative n_{eff}. The reason is that N_D changes the complex permittivity of silicon, which affects the wave refraction and transmission at the interfaces of the ferrite and silicon. Consequently, it changes the effective index of the composite.

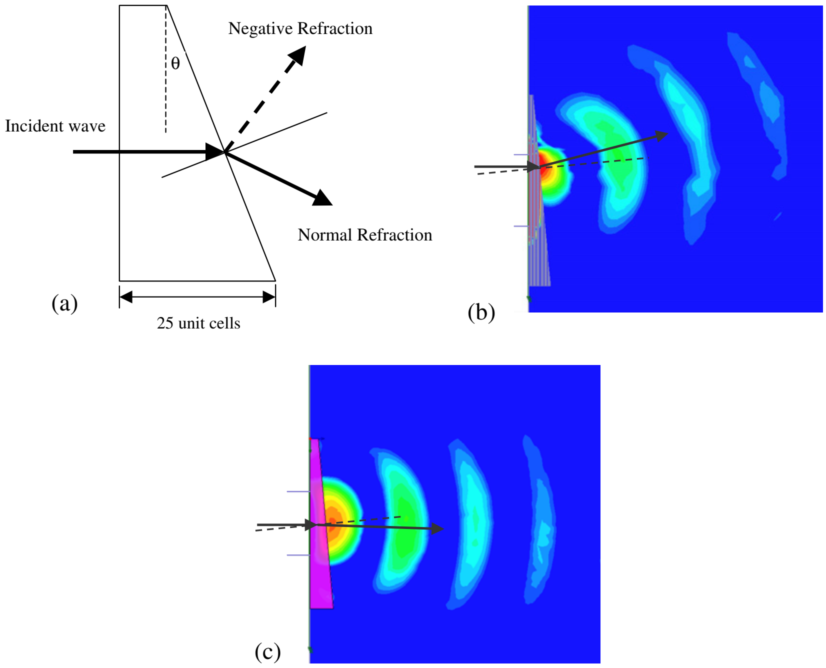

Figure 3. Simulations of electric field distribution at 4 GHz. (a) Schematic diagram of the pyramid in our simulation. The pyramid with angle $\theta = 5°$ is composed by BaCo–Si multilayers, where the thickness ratio is 0.2 and unit cell length is 0.14 mm. With $N_D = 10^{16}$ cm^{-3}, the effective refraction index is about $n = -6$. (b) Field distribution of the wave refracted from the pyramid, which shows negative refraction. (c) Field distribution of the wave refracted from a pyramid made of uniform medium with positive refraction index $n = +6$. To reduce the computing space, a lossless dielectric with index $n = +3$ is used outside the pyramid in the simulations.

(This figure is in colour only in the electronic version)

To validate the proposed composite with a negative index of refraction, we perform electromagnetic simulations on a pyramid made of the composite. Figure 3(a) gives a schematic diagram of the pyramid in our simulations. The simulation is performed at 4 GHz with $N_D = 10^{16}$ cm^{-3}, at which with the parameters of the BaCo and doped Si shown above the composite has an effective index of refraction of about $n = -6$, as can be found in the curve within figure 2. Simulation results are shown in figure 3(b). We can find that the beam incident on the interface of the prism and background medium refracts to the same side of the normal as the incident wave. This demonstrates that the index of refraction of the composite is negative. As a comparison, figure 3(c) gives the simulation result for the prism with a positive index $n = +6$. It can be find that, for normal materials, the refraction ray lies on a different side of the normal to the incident ray.

In addition to the effect of N_D, the unit cell configuration, i.e. the unit cell length d and the thickness ratio $r = d_1/d_2$ of silicon and ferrite layers, also affects the negative n_{eff}. Figure 4(a) plots the n_{eff} as a function of the thickness ratio at 4 GHz. In the calculation, we take the doping content of silicon as $N_D = 10^{16}$ cm^{-3}, which gives a conductivity of about 177 S m^{-1} according to reference [18], which in turn gives a complex permittivity $\varepsilon_r = 11.9$–796 j and penetration depth δ of doped silicon layers of about 0.6 mm. At 4 GHz, the permeability of the BaCo is about $\mu_r = -3.1$–6.1 j. We suppose that the thickness of the doped silicon layers, d_1, is much less than the penetration depth ($d_1/\delta < 0.1$), in order to let electromagnetic (EM)

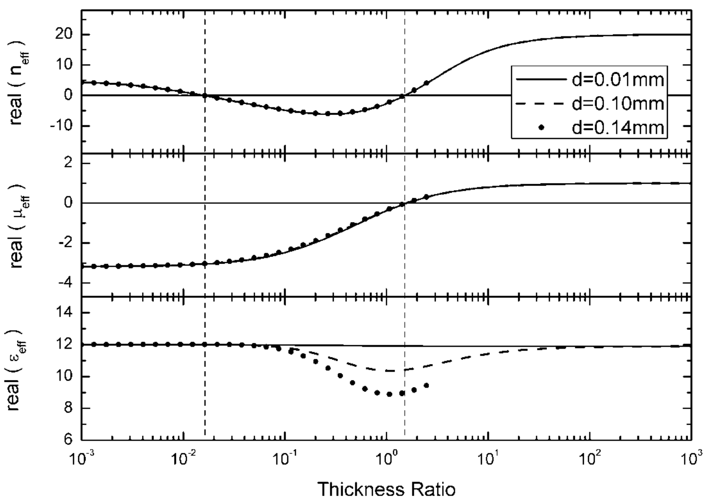

Figure 4. Thickness ratio dependence of the (a) effective index of refraction, (b) effective permeability μ_{eff}, and (c) effective permittivity ε_{eff} of the composites with different unit cell length.

waves pass through it. It can be found in the figure that the magnitude of n_{eff} varies with the thickness ratio, but depends weakly on the length of the unit cell. For a certain thickness ratios, n_{eff} has negative values. With equations (5)–(8), we have calculated the effective permeability μ_{eff} and permittivity ε_{eff} under criteria that the length of the unit cell, d/λ, is less than 0.1 and $d_1/\delta < 0.1$. The results are shown in figure 4(b), (c). The effective permeability μ_{eff} depends only on the thickness ratio, and the effective permittivity has little dependence on the unit cell length. Compared with these figures, it is obvious that the negative n_{eff} originates from negative μ_{eff}, and the multilayer is an only-mu-negative NIM.

We have shown the effects of the complex permittivity of silicon and unit cell configuration on the effective refraction index, permeability and permittivity using a numerical method. In the long-wave approximation, an explicit expression can be obtained. Suppose that βd, $k_1 d_1$ and $k_2 d_2$ are very small, expanding equation (5) and keeping quadratic terms only, we have

$$n_{\text{eff}}^2 = \left(\frac{2r\varepsilon_1 + \varepsilon_2}{1+2r}\right)\left(\frac{2r\mu_1 + \mu_2}{1+2r}\right) = \varepsilon_{\text{eff}}\mu_{\text{eff}}, \tag{9}$$

where r is the thickness ratio, and ε_{eff} and μ_{eff} are the effective permittivity and permeability of the composite, respectively. Equation (9) indicates that the thickness ratio alters the contribution of the ferrite and semiconductor through the terms ε_{eff} and μ_{eff}. For very small or very big thickness ratios, ε_{eff} and μ_{eff} are dominated by either the ferrite or the semiconductor, respectively, and then n_{eff} is always positive for these two extremes. If the ferrite has negative permeability and the ratio is not too small so that $\mu_{\text{eff}} \approx \mu_2$ and the phase of ε_{eff} is big enough, then the phase of the product $\varepsilon_{\text{eff}}\mu_{\text{eff}}$ is in the second quadrant, as illustrated in figure 5. Consequently, the index of refraction $n_{\text{neff}} = \sqrt{\varepsilon_{\text{eff}}\mu_{\text{eff}}}$ is negative. As can also be found in equation (9), the index n_{eff} is independent of the unit cell length. Therefore, the unit cell length could be small so long as the values of ε and μ of the ferrite and the semiconductor are not altered due to finite-size effect. Usually, ε and μ of a particle are the same as those of bulk materials if the particle size is greater than several hundred nanometres. So, the length of the unit cell of the composite could be of the order of a micrometre, which enables the fabrication of NIMs based on thin films, which is much more convenient than the structured composites. Meanwhile, the tunable character of the doping content of semiconductors and

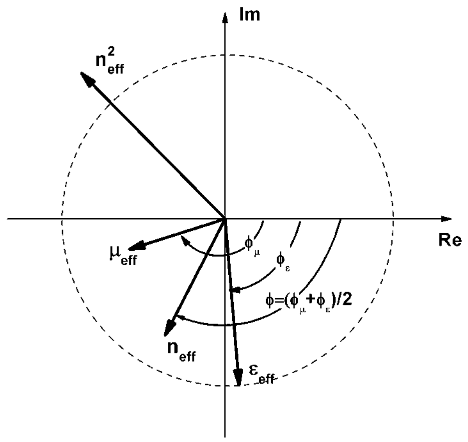

Figure 5. Schematic diagram of n_{eff}, $\varepsilon_{\mathrm{eff}}$ and μ_{eff} on the complex plane. If μ_{eff} has a negative real part, then the composite has a negative index as long as $\varepsilon_{\mathrm{eff}}$ is close to the imaginary axis, even though its real part is positive.

the sufficiently usable thickness ratios make the fabrication of NIMs not as difficult as that of a metallic film based composite. In addition, since the length of unit cell is of the order of a micrometre, which is much smaller than the wavelength λ in the composite at microwave frequencies, the composite can be considered as a uniform medium. Therefore, the proposed configuration provides a possible way to fabricate uniform negative index materials. It may be noticed that this only-mu-negative NIM is lossy. However, since this NIM can be in the form of thin films, its thickness could be much smaller than the effective penetration depth, and whole damping could be lower than the metamaterial. Meanwhile, there are some ways to reduce the loss effectively, such as applying a magnetic field, choosing an appropriate configuration, etc, which we are currently working on.

In conclusion, we have studied the index of refraction of a periodic layered BaCo–Si composite, and demonstrated that the composite can be a negative index material under certain conditions. The magnitude of the effective negative index of refraction and its frequency band can be controlled by the doping content of the semiconductor and the composite configuration. We have calculated the effective permittivity and permeability of the composite and found that the negative refraction index arises from the negative permeability, which indicates that the composite is an only-mu-negative NIM. In the long-wave approximation, we have derived an explicit expression for the refraction index and shown that the length of the unit cell can be of the order of micrometres, therefore it is possible to fabricate uniform negative refraction index materials with the proposed configuration.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (under project 60471020), the National Basic Research Program (973) of China (under Project 2004CB719805), and the US DEPSCoR W911NF-04-1-0264.

References

[1] Shelby R A, Smith D R and Schultz S 2001 *Science* **296** 77
[2] Veselago V G 1968 *Sov. Phys.—Usp.* **10** 509

[3] Ramakrishna S A and Pendry J B 2002 *J. Mod. Opt.* **49** 1747
[4] Pendry J B 2000 *Phys. Rev. Lett.* **85** 3966
[5] Grbic A and Eleftheriades G V 2004 *Phys. Rev. Lett.* **92** 117403
[6] Parimi P V, Lu W T, Vodo P, Sokoloff J, Derov J S and Sridhar S 2004 *Phys. Rev. Lett.* **92** 127401
[7] Chen H, Ran L, Wang D, Huangfu J, Jiang Q and Kong J A 2006 *Appl. Phys. Lett.* **88** 031908
[8] Bayindir M, Aydin K, Ozbay E, Markos P and Soukoulis C M 2002 *Appl. Phys. Lett.* **81** 120
[9] Smith D R, Schultz S, Markos P and Soukoulis C M 2002 *Phys. Rev.* B **65** 195104
[10] Ueda T and Tsutsumi M 2005 *IEEE Trans. Magn.* **41** 3532
[11] Wu R X 2005 *J. Appl. Phys.* **97** 076105
[12] Pimenov A, Loidl A, Przyslupski P and Dabrowski B 2005 *Phys. Rev. Lett.* **95** 247009
[13] Born M and Wolf E 1999 *Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light* 7th edn (New York: Cambridge University Press)
[14] Cao Z Q 2002 *Transfer Matrix Method in Guided Wave Optics* (Shanghai: Shanghai Jiaotong University Press) (in Chinese)
[15] Zhang K Q and Li D J 1999 *Electromagnetic Theory for Microwaves and Optoelectronics* (Berlin: Springer)
[16] Dowling J P and Bowden C M 1994 *J. Mod. Opt.* **41** 345
[17] Kong J A 2005 *Electromagnetic Wave Theory* (Cambridge: EMW Publishing)
[18] Sze S M and Irvin J C 1968 *Solid-State Electron.* **11** 599